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Plane Symmetric Viscous Fluid Cosmological Models with Varying Λ-Term

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Abstract Plane symmetric viscous fluid cosmological models of the universe with a variable cosmological term are investigated. The viscosity coefficient of bulk viscous fluid is assumed to be a power function of mass density whereas the coefficient of shear viscosity is to be proportional to rate of expansion in the model. We have also obtained a special model in which the shear viscosity is assumed to be zero. The cosmological constant Λ is found to be a decreasing function of time and a positive which is supported by results from recent supernovae Ia observations. Some physical and geometric properties of the models are also discussed.

Keywords Cosmology · Viscous models · Variable cosmological term

1 Introduction

In modern cosmological theories, a dynamic cosmological term $\Lambda(t)$ remains a focal point of interest as it solves the cosmological constant problem in a natural way. There are significant observational evidence for the detection of Einstein's cosmological constant, Λ or a

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component of material content of the universe that varies slowly with time and space to act like Λ. A wide range of observations now compellingly suggest that the universe possesses a non-zero cosmological term [16]. In the context of quantum field theory, a cosmological term corresponds to the energy density of vacuum. The birth of the universe has been attributed to an excited vacuum fluctuation triggering off an inflationary expansion followed by the super-cooling. The release of locked up vacuum energy results in subsequent reheating. The cosmological term, which is measure of the energy of empty space, provides a repulsive force opposing the gravitational pull between the galaxies. If the cosmological term exists, the energy it represents counts as mass because mass and energy are equivalent. If the cosmological term is large enough, its energy plus the matter in the universe could lead to inflation. Unlike standard inflation, a universe with a cosmological term would expand faster with time because of the push from the cosmological term [10]. Some of the recent discussions on the cosmological constant "problem" and on cosmology with a time-varying cosmological constant are given by Carroll, Press and Turner [8]; Sahni and Starobinsky [47]; Peebles [26]; Ratra and Peebles [43]; Padmanabhan [23] and Pradhan et al. [40, 41]. Recently, Belinchon and Chakrabarty [5] have studied in detail a perfect fluid cosmological model with time-varying "constants" using dimensional analysis and the symmetry method.

Recent observations by Perlmutter et al. [27–29] and Riess et al. [44] strongly favour a significant and a positive value of Λ with magnitude $\Lambda(G\hbar/c^3) \approx 10^{-123}$. Their study is based on more than 50 type Ia supernovae with red-shifts in the range $0.10 \leq z \leq 0.83$ and these suggest Friedmann models with negative pressure matter such as a cosmological constant (Λ), domain walls or cosmic strings [11, 12, 51]. Recently, Carmeli and Kuzmenko [9] and Behar and Carmeli [4] have shown that the cosmological relativistic theory predicts the value for cosmological constant $\Lambda = 1.934 \times 10^{-35}$ s^{-2}. This value of "Λ" is in excellent agreement with the recent estimates of the High-Z Supernova Team and Supernova Cosmological Project [11, 12, 27–29, 44, 49]. Riess et al. [45] have recently presented an analysis of 156 SNe including a few at $z > 1.3$ from the Hubble Space Telescope (HST) "GOOD ACS" Treasury survey. They conclude to the evidence for present acceleration $q_0 < 0$ ($q_0 \approx -0.7$). Observations [15, 45] of Type Ia Supernovae (SNe) allow us to probe the expansion history of the universe leading to the conclusion that the expansion of the universe is accelerating.

The distribution of matter can be satisfactorily described by a perfect fluid due to the large scale distribution of galaxies in our universe. However, observed physical phenomena such as the large entropy per baryon and the remarkable degree of isotropy of the cosmic microwave background radiation, suggest analysis of dissipative effects in cosmology. Furthermore, there are several processes which are expected to give rise to viscous effects. These are the decoupling of neutrinos during the radiation era and the decoupling of radiation and matter during the recombination era. Bulk viscosity is associated with the GUT phase transition and string creation. Misner [18, 19]; Weinberg [52]; Nightingale [22]; Heller and Klimek [13]; Murphy [20]; Roy and Prakash [46]; Bali and Jain [1–3]; Kalyani and Singh [14] have investigated the effect of bulk viscosity on the evolution of cosmological models. Recently, this phenomenon is studied by Pradhan et al. [30–42] and Belinchon [6]. This motivates to study cosmological bulk viscous fluid model.

Recently, Singh and Yadav [50] have investigated Bianchi type I viscous fluid cosmological models in general relativity. Motivated by the situations discussed above, in this paper, we shall focus on the problem with varying cosmological constant in presence of bulk fluid and shear viscous fluid in an expanding universe. We revisit and extend Singh and Yadav's work by including varying cosmological constant and the coefficient of bulk viscosity as function of time. This paper is organized as follows: The metric and the field equations are presented in Sect. 2. In Sect. 3, we deal with the general solution of the field equations by considering expansion as proportional to eigenvalue σ_2^2 of the shear tensor and shear viscosity as proportional to θ. The bulk viscosity is assumed to be simple power of energy density $\xi = \xi_0 \rho^n$. In Sects. 3.1 and 3.2, we consider the solutions for $n = 0$ and $n = 1$. Section 4 includes the solution in absence of shear viscosity. Finally in Sect. 5 we mention our main conclusions.

2 The Metric and Field Equations

We consider the plane symmetric metric in the form

$$ds^2 = A^2(dx^2 - dt^2) + B^2 dy^2 + C^2 dz^2, \tag{1}$$

where A, B and C are functions of t alone.

The Einstein's field equations (in gravitational units $c = 1$, $G = 1$) read as

$$R_i^j - \frac{1}{2} R g_i^j + \Lambda g_i^j = -8\pi T_i^j, \tag{2}$$

where R_i^j is the Ricci tensor; $R = g^{ij} R_{ij}$ is the Ricci scalar; and T_i^j is the stress energy-tensor in the presence of bulk stress given by

$$\begin{aligned} T_i^j &= (\rho + p) v_i v^j + p g_i^j - \eta (v_{i;}^j + v^j{}_{;i} + v^j v^l v_{i;l} + v_i v^l v^j{}_{;l}) \\ &\quad - \left(\xi - \frac{2}{3}\eta\right)\theta (g^j{}_i + v_i v^j). \end{aligned} \tag{3}$$

Here ρ, p, η and ξ are the energy density, isotropic pressure, coefficient of shear viscosity and bulk viscous coefficient respectively and v_i is the flow vector satisfying the relations

$$g_{ij} v^i v^j = -1. \tag{4}$$

The semicolon (;) indicates covariant differentiation. We choose the coordinates to be comoving, so that

$$v^1 = 0 = v^2 = v^3, \qquad v^4 = \frac{1}{A}. \tag{5}$$

The Einstein's field equations (2) for the line element (1) has been set up as

$$\begin{aligned} &-8\pi\left[p - 2\eta \frac{A_4}{A^2} - \left(\xi - \frac{2}{3}\eta\right)\theta\right] \\ &\quad = \frac{1}{A^2}\left[\frac{B_{44}}{B} + \frac{C_{44}}{C} + \frac{B_4 C_4}{BC} - \frac{A_4 B_4}{AB} - \frac{A_4 C_4}{AC}\right] + \Lambda, \end{aligned} \tag{6}$$

$$-8\pi\left[p - 2\eta \frac{B_4}{AB} - \left(\xi - \frac{2}{3}\eta\right)\theta\right] = \frac{1}{A^2}\left[\frac{C_{44}}{C} + \frac{A_{44}}{A} - \frac{A_4^2}{A^2}\right] + \Lambda, \tag{7}$$

$$-8\pi\left[p - 2\eta \frac{C_4}{AC} - \left(\xi - \frac{2}{3}\eta\right)\theta\right] = \frac{1}{A^2}\left[\frac{B_{44}}{B} + \frac{A_{44}}{A} - \frac{A_4^2}{A^2}\right] + \Lambda, \tag{8}$$

$$8\pi\rho = \frac{1}{A^2}\left[\frac{B_4 C_4}{BC} + \frac{A_4 B_4}{AB} + \frac{A_4 C_4}{AC}\right] + \Lambda, \tag{9}$$

where the suffix 4 at the symbols A, B and C denotes ordinary differentiation with respect to t and θ is the scalar of expansion given by

$$\theta = v^i{}_{;i}. \tag{10}$$

3 Solution of the Field Equations

We have revisited the solutions obtained by Singh and Yadav [50]. Equations (6–9) are four independent equations in eight unknowns A, B, C, ρ, p, η, ξ and Λ. For the complete determinacy of the system, we need four extra conditions. The research on exact solutions is based on some physically reasonable restrictions used to simplify the Einstein equations.

To get a determinate solution we first assume that the expansion (θ) in the model is proportional to the eigenvalue σ_2^2 of the shear tensor. This condition leads to

$$B = AC, \tag{11}$$

without any loss of generality. Secondly we assume that the coefficient of shear viscosity η is proportional to the rate of expansion in the model i.e. $\eta \propto \theta$ which together with (11) leads to

$$\eta = 2\ell \frac{B_4}{AB}, \tag{12}$$

where ℓ is proportionality constant.

From (6–8), we obtain

$$\left(\frac{A_4}{A}\right)_4 + \left(\frac{A_4}{A}\right)\left(\frac{B_4}{B} + \frac{C_4}{C}\right) - \frac{B_{44}}{B} - \frac{B_4 C_4}{BC} = 16\pi\eta A\left(\frac{B_4}{B} - \frac{A_4}{A}\right), \tag{13}$$

$$\frac{(CB_4 - BC_4)_4}{(CB_4 - BC_4)} = -16\pi\eta A. \tag{14}$$

Using (11) and (12) in (13) reduces to

$$C_4 = \beta B^{\alpha}, \tag{15}$$

where $\alpha = -(32\pi\ell + 1)$ and β is an integrating constant. Equation (14) together with (12) leads to

$$C^2\left(\frac{B}{C}\right)_4 = MB^{(\alpha+1)}, \tag{16}$$

where M is an integrating constant. From (15) and (16), we obtain

$$\frac{C_4}{C} = k\frac{\left(\frac{B}{C}\right)_4}{\left(\frac{B}{C}\right)}, \tag{17}$$

which leads to

$$\frac{\mu_4}{\mu} = (2k+1)\frac{\nu_4}{\nu}, \tag{18}$$

where $BC = \mu$, $\frac{B}{C} = \nu$ and $k = \frac{\beta}{M}$. Equation (16) leads to

$$\frac{\nu_4}{\nu^{\frac{\alpha+3}{2}}} = M\mu^{\frac{\alpha-1}{2}}. \tag{19}$$

On integrating (18), we obtain

$$\mu = N\nu^{2k+1}, \tag{20}$$

where N is a constant of integration. Equations (19) and (20) lead to

$$\frac{\nu_4}{\nu^m} = a, \tag{21}$$

where $m = \alpha k + \alpha - k + 1$ and $a = MN^{\frac{\alpha-1}{2}}$. Integrating (21), we have

$$\nu = (1-m)^{\frac{1}{1-m}}(at+b)^{\frac{1}{1-m}}, \tag{22}$$

where b is an integrating constant. From (20) and (22), we have

$$\mu = N[(1-m)^{\frac{2k+1}{1-m}}(at+b)^{\frac{2k+1}{1-m}}]. \tag{23}$$

Thus from (22) and (23), we obtain

$$B = \sqrt{\mu\nu} = \sqrt{N}(1-m)^{\frac{k+1}{1-m}}(at+b)^{\frac{k+1}{1-m}}, \tag{24}$$

$$C = \sqrt{\frac{\mu}{\nu}} = \sqrt{N}(1-m)^{\frac{k}{1-m}}(at+b)^{\frac{k}{1-m}}. \tag{25}$$

Using (24) and (25) in (11) reduces to

$$A = (1-m)^{\frac{1}{1-m}}(at+b)^{\frac{1}{1-m}}. \tag{26}$$

After using the transformation of coordinates

$$\begin{aligned} (at+b) &= T, \\ (1-m)^{\frac{1}{1-m}}x &= X, \\ \sqrt{N}(1-m)^{\frac{k+1}{1-m}}y &= Y, \\ \sqrt{N}(1-m)^{\frac{k}{1-m}}z &= Z, \end{aligned} \tag{27}$$

the metric (1) reduces to the form

$$ds^2 = T^{\frac{2}{1-m}}dX^2 - \frac{(1-m)^{\frac{2}{1-m}}}{a^2}T^{\frac{2}{1-m}}dT^2 + T^{\frac{2(k+1)}{1-m}}dY^2 + T^{\frac{2k}{1-m}}dZ^2. \tag{28}$$

The pressure and density for the model (28) are given by

$$\begin{aligned} 8\pi p = {} & \frac{a^2}{3}[(32\pi\ell - 3)k^2 - (32\pi\ell + 6)k - (64\pi\ell + 3)](1-m)^{\frac{2(m-2)}{1-m}}T^{\frac{2(m-2)}{1-m}} \\ & + a^2(k+2)(1-m)^{\frac{m-3}{1-m}}T^{\frac{2m-4}{1-m}} + 16\pi a(k+1)\xi(1-m)^{\frac{m-2}{1-m}}T^{\frac{m-2}{1-m}} - \Lambda, \end{aligned} \tag{29}$$

$$8\pi\rho = a^2(k^2+3k+1)(1-m)^{\frac{2(m-2)}{1-m}}T^{\frac{2(m-2)}{1-m}} + \Lambda. \tag{30}$$

For the specification of ξ, we assume that the fluid obeys an equation of state of the form

$$p = \gamma\rho, \tag{31}$$

where $\gamma(0 \le \gamma \le 1)$ is constant. Thus, given $\xi(t)$ we can solve for the cosmological parameters. In most of the investigation involving bulk viscosity it is assumed to be a simple power function of the energy density [17, 24, 25, 48, 54]

$$\xi(t) = \xi_0\rho^n, \tag{32}$$

where ξ_0 and n are constants. For small density, n may even be equal to unity as used in Murphy's work [20] for simplicity. If $n = 1$, (32) may correspond to a radiative fluid [53]. Near the big bang, $0 \le n \le \frac{1}{2}$ is a more appropriate assumption [7] to obtain realistic models.

For simplicity and realistic models of physical importance, we consider the following two cases ($n = 0, 1$):

3.1 Model I: Solution for $\xi = \xi_0$

When $n = 0$, (32) reduces to $\xi = \xi_0 =$ constant. Hence in this case (29), with the use of (30) and (31), leads to

$$\begin{aligned} 8\pi(1+\gamma)\rho &= a^2\left[\frac{1}{3}\{(32\pi\ell)k^2 - 3(\pi\ell-1)k - (64\pi\ell)\}\right](1-m)^{\frac{2(m-2)}{1-m}}T^{\frac{2(m-2)}{1-m}} \\ &\quad + a^2(k+2)(1-m)^{\frac{(m-3)}{1-m}}T^{\frac{2(m-2)}{1-m}} \\ &\quad + 32\pi a\xi_0(k+1)(1-m)^{\frac{(m-2)}{1-m}}T^{\frac{(m-2)}{1-m}}. \end{aligned} \tag{33}$$

Eliminating $\rho(t)$ between (30) and (33), we have

$$\begin{aligned} (1+\gamma)\Lambda &= a^2\left[\frac{1}{3}\{(32\pi\ell-3\gamma-3)k^2 - 3(\pi\ell+3\gamma+2)k - (64\pi\ell-3\gamma+3)\}\right] \\ &\quad \times (1-m)^{\frac{2(m-2)}{1-m}}T^{\frac{2(m-2)}{1-m}} + a^2(k+2)(1-m)^{\frac{(m-3)}{1-m}}T^{\frac{2(m-2)}{1-m}} \\ &\quad + 32\pi a\xi_0(k+1)(1-m)^{\frac{(m-2)}{1-m}}T^{\frac{(m-2)}{1-m}}. \end{aligned} \tag{34}$$

From (33), we note that $\rho(t)$ is a decreasing function of time and $\rho > 0$ for all times. Figure 1 shows this behaviour of energy density.

The behaviour of the universe in this model will be determined by the cosmological term Λ; this term has the same effect as a uniform mass density $\rho_{\text{eff}} = -\Lambda/4\pi G$, which is constant in space and time. A positive value of Λ corresponds to a negative effective mass density (repulsion). Hence, we expect that in the universe with a positive value of Λ, the expansion will tend to accelerate; whereas in the universe with negative value of Λ, the expansion will slow down, stop and reverse. From (34), we see that the cosmological term Λ is a decreasing function of time and it approaches a small positive value as time increase more and more. From Fig. 2 we note this behaviour of Λ. Recent cosmological observations [11, 12, 27–29, 44, 45, 49] suggest the existence of a positive cosmological constant Λ with the magnitude $\Lambda(G\hbar/c^3) \approx 10^{-123}$. These observations on magnitude and red-shift of type Ia supernova suggest that our universe may be an accelerating one with induced cosmological density through the cosmological Λ-term. Thus, our model is consistent with the results of recent observations.

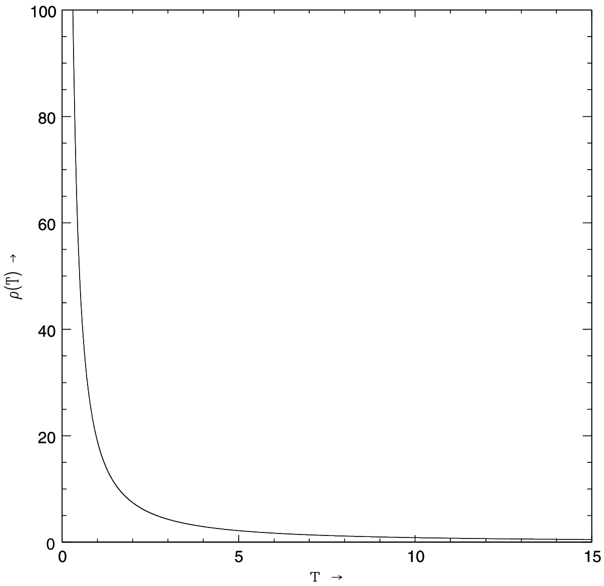

Fig. 1 The plot of energy density $\rho(t)$ vs. time

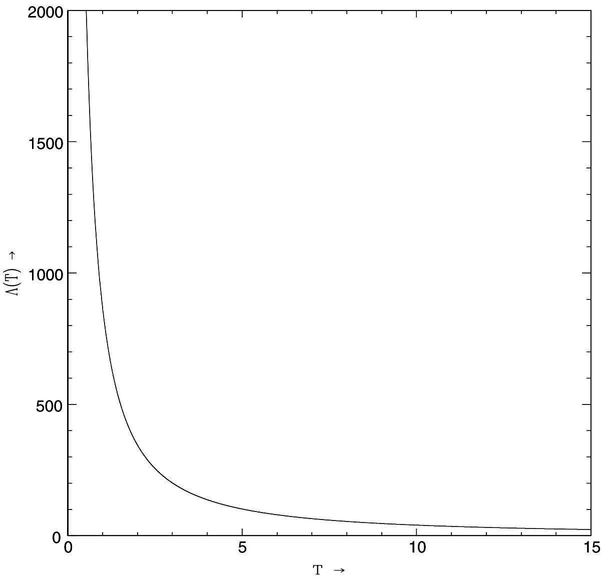

Fig. 2 The plot of cosmological term $\Lambda(t)$ vs. time

3.2 Model I: Solution for $\xi = \xi_0 \rho$

When $n = 1$, (31) reduces to $\xi = \xi_0 \rho$. Hence in this case (29), with the use of (30) and (31), leads to

$$8\pi\rho = \frac{a^2(1-m)^{\frac{m-3}{1-m}} T^{\frac{2(m-2)}{1-m}} [k + 2 + \{\frac{32\pi\ell(k^2-2)}{3} - k(\pi\ell - 1)\}(1-m)^{-1}]}{[(1+\gamma) - 4a(k+1)\xi_0(1-m)^{\frac{m-2}{1-m}} T^{\frac{m-2}{1-m}}]}. \tag{35}$$

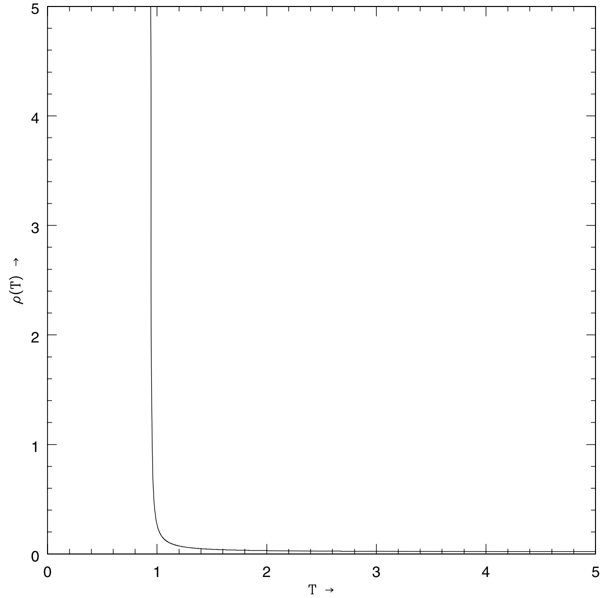

Fig. 3 The plot of energy density $\rho(t)$ vs. time

Eliminating $\rho(t)$ between (30) and (35), we have

$$\Lambda = \frac{a^2(1-m)^{\frac{m-3}{1-m}} T^{\frac{2(m-2)}{1-m}} [k+2+\{\frac{32\pi\ell(k^2-2)}{3} - k(\pi\ell-1)\}(1-m)^{-1}]}{[(1+\gamma) - 4a(k+1)\xi_0(1-m)^{\frac{m-2}{1-m}} T^{\frac{m-2}{1-m}}]} - a^2(k^2+3k+1)(1-m)^{\frac{2(m-2)}{1-m}} T^{\frac{2(m-2)}{1-m}}. \tag{36}$$

From (35), we observe that $\rho(t)$ is a decreasing function of time and $\rho > 0$ for all times. Figure 3 shows this behaviour of energy density. From (36), we note that the cosmological term Λ is a decreasing function of time and it approaches a small positive value as time increase more and more. From Fig. 4 we note the same character of Λ. This is consistent with recent observations [11, 12, 27–29, 44, 45, 49].

We shall now give the expressions for kinematic quantities and components of conformal curvature tensor. With regard to the kinematical properties of the velocity vector v^i in the metric (28), a straightforward calculation leads to the expressions for expansion (θ) and shear (σ) of the fluid:

$$\theta = 2a(k+1)(1-m)^{\frac{m-2}{1-m}} T^{\frac{m-2}{1-m}}, \tag{37}$$

$$\sigma = a\left[\frac{2}{3}(k^2-k+1)\right]^{\frac{1}{2}} (1-m)^{\frac{m-2}{1-m}} T^{\frac{m-2}{1-m}}. \tag{38}$$

Equations (37) and (38) lead to

$$\frac{\sigma}{\theta} = \frac{(k^2-k+1)^{\frac{1}{2}}}{\sqrt{6}(k+1)}. \tag{39}$$

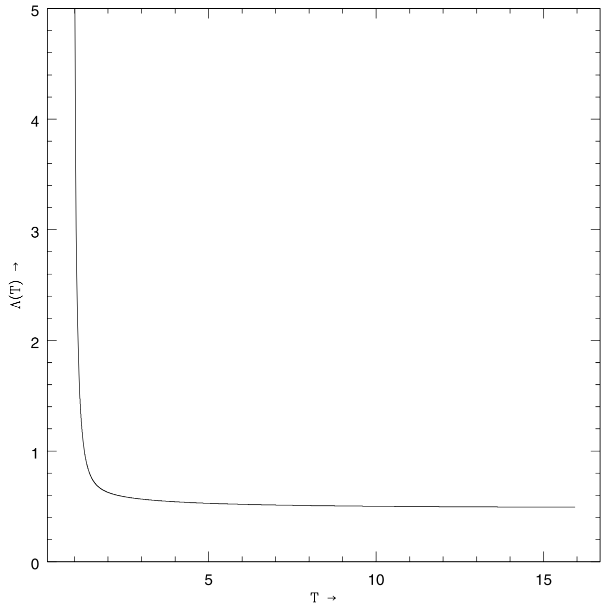

Fig. 4 The plot of cosmological term $\Lambda(t)$ vs. time

The rotation ω and acceleration are identically zero. The non-vanishing physical components of the components of conformal curvature tensor are given by

$$C^{12}{}_{12} = \frac{a^2}{6}(3k-2)(1-m)^{\frac{2(m-2)}{1-m}} T^{\frac{2(m-2)}{1-m}} + \frac{a^2}{6}(k-2)(1-m)^{\frac{(m-3)}{1-m}} T^{\frac{2(m-2)}{1-m}}, \tag{40}$$

$$C^{13}{}_{13} = \frac{a^2}{6}(1-3k)(1-m)^{\frac{2(m-2)}{1-m}} T^{\frac{2(m-2)}{1-m}} + \frac{a^2}{6}(k+1)(1-m)^{\frac{(m-3)}{1-m}} T^{\frac{2(m-2)}{1-m}}, \tag{41}$$

$$C^{14}{}_{14} = \frac{a^2}{6}(1-m)^{\frac{2(m-2)}{1-m}} T^{\frac{2(m-2)}{1-m}} + \frac{a^2}{6}(1-2k)(1-m)^{\frac{(m-3)}{1-m}} T^{\frac{2(m-2)}{1-m}}. \tag{42}$$

Here we find that

$$C^{12}{}_{12} + C^{13}{}_{13} + C^{14}{}_{14} = 0. \tag{43}$$

The models represent shearing, non-rotating and Petrov Type I non-degenerate universe in general, in which the flow is geodetic. The cosmological model (28) starts expanding with a big bang singularity at $T = 0$ when $m > 2$. The models start expanding at $T = 0$ and reach on expanding till $T \to \infty$ when $1 < m < 2$. When $T \to 0$ then $\rho \to \infty$ and $p \to \infty$ if $1 < m < 2$. When $T \to \infty$ then $\rho \to 0$, $p \to 0$ if $1 < m < 2$. Since $\lim_{T\to 0} \frac{\sigma}{\theta} \neq 0$, the models do not approach isotropy for large values of T. There is a Point Type singularity in the models at $T = 0$ when $m < 1$ and $k > 0$.

4 Solutions in Absence of Shear Viscosity

When $\eta \to 0$, then the metric (28) leads to

$$ds^2 = T^{\frac{2}{1+2k}} dX^2 - \frac{N^2}{M^2}(1+2k)^{\frac{2}{1+2k}} T^{\frac{2}{1+2k}} dT^2 + T^{\frac{2(1+k)}{1+2k}} dY^2 + T^{\frac{2k}{1+2k}} dZ^2. \quad (44)$$

The pressure and density for the model (44) are given by

$$8\pi p = \frac{M^2}{N^2}[P - R]T^{-\frac{4(k+1)}{(1+2k)}} + 16\pi\xi S T^{-\frac{2(k+1)}{(1+2k)}} - \Lambda, \quad (45)$$

$$8\pi\rho = \frac{M^2}{N^2} Q T^{-\frac{4(k+1)}{(1+2k)}} + \Lambda, \quad (46)$$

where

$$\begin{aligned} P &= (k+2)(1+2k)^{-\frac{(2k+3)}{(1+2k)}}, \\ Q &= (k^2+3k+1)(1+2k)^{-\frac{4(k+1)}{(1+2k)}}, \\ R &= (k^2+2k-1)(1-2k)^{-\frac{4(k+1)}{(1+2k)}}, \\ S &= \frac{M}{N}(k+1)(1+2k)^{-\frac{2(k+1)}{(1+2k)}}. \end{aligned} \quad (47)$$

4.1 Model I: Solution for $\xi = \xi_0$

When $n = 0$, (32) reduces to $\xi = \xi_0$ (constant). Hence in this case (45), with the use of (31) and (46), leads to

$$8\pi(1+\gamma)\rho = \frac{M^2}{N^2}[P + Q - R] T^{-\frac{4(k+1)}{(1+2k)}} + 16\pi\xi_0 S T^{-\frac{2(1+k)}{(1+2k)}}. \quad (48)$$

Eliminating $\rho(t)$ between (46) and (48), we have

$$(1+\gamma)\Lambda = \frac{M^2}{N^2}[P - R - \gamma Q]T^{-\frac{4(k+1)}{(1+2k)}} + 16\pi\xi_0 S T^{-\frac{2(1+k)}{(1+2k)}}. \quad (49)$$

From (48), we see that $\rho(t)$ is a decreasing function of time and $\rho > 0$ for all times. Figure 5 shows this behaviour of energy density. From (49), we note that the cosmological term Λ is a decreasing function of time and it approaches a small positive value as time increase more and more. From Fig. 6 we note the same character of Λ. This is consistent with recent observations [11, 12, 27–29, 44, 45, 49].

4.2 Model II: Solution for $\xi = \xi_0\rho$

When $n = 1$, (32) reduces to $\xi = \xi_0\rho$. Hence in this case (45), with the use of (29) and (46), leads to

$$8\pi\rho = \frac{\frac{M^2}{N^2}[P - R + Q] T^{-\frac{4(1+k)}{(1+2k)}}}{(1+\gamma) - 2S\xi_0 T^{-\frac{2(k+1)}{1+2k}}}. \quad (50)$$

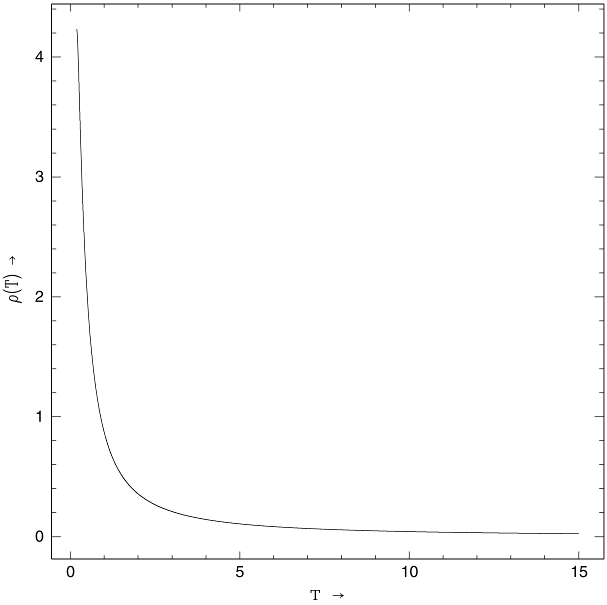

Fig. 5 The plot of energy density $\rho(t)$ vs. time

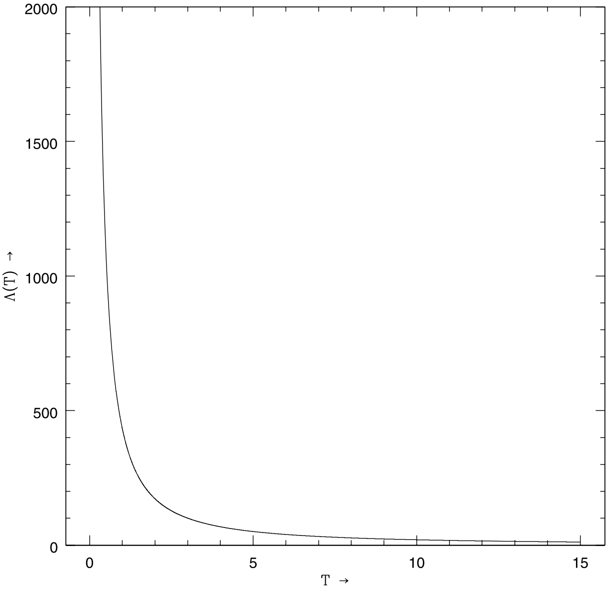

Fig. 6 The plot of cosmological term $\Lambda(t)$ vs. time

Eliminating $\rho(t)$ between (46) and (50), we have

$$\Lambda = \frac{\frac{M^2}{N^2}\,[P - R + Q]\,T^{-\frac{4(1+k)}{(1+2k)}}}{(1+\gamma) - 2S\xi_0 T^{-\frac{2(k+1)}{1+2k}}} - \frac{M^2}{N^2}Q. \tag{51}$$

In this case the value of energy density $\rho(t)$ and cosmological term $\Lambda(t)$, for wide range of parameters, have been examined. We find in all these cases ρ and Λ both remain negative throughout the evolution. Hence, the models in case $\xi = \xi_0 \rho^n$ $(n \geq 1)$ may not be physical in this set up. So it will not be pursue further.

With regard to the kinematical properties of the velocity vector v^i in the metric (44), a straightforward calculation leads to the expressions for expansion (θ) and shear (σ) of the fluid:

$$\theta = 2\frac{M}{N}(k+1)(1+2k)^{-\frac{2(k+1)}{1+2k}} T^{-\frac{2(k+1)}{1+2k}}, \tag{52}$$

$$\sigma = \frac{M}{N}\sqrt{\frac{2}{3}}(k^2-k+1)^{\frac{1}{2}}(k+1)(1+2k)^{-\frac{2(k+1)}{1+2k}} T^{-\frac{2(k+1)}{1+2k}}. \tag{53}$$

Equations (52) and (53) lead to

$$\frac{\sigma}{\theta} = \frac{(k^2-k+1)^{\frac{1}{2}}}{\sqrt{6}(k+1)}. \tag{54}$$

The rotation ω and acceleration are identically zero. The non-vanishing physical components of the components of conformal curvature tensor are given by

$$C^{12}{}_{12} = \frac{M^2}{6N^2}(3k-2)(1+2k)^{-\frac{4(k+1)}{1+2k}} T^{-\frac{4(k+1)}{1+2k}} + \frac{M^2}{6N^2}(k-2)(1+2k)^{-\frac{(2k+3)}{1+2k}} T^{-\frac{4(k+1)}{1+2k}}, \tag{55}$$

$$C^{13}{}_{13} = \frac{M^2}{6N^2}(1-3k)(1+2k)^{-\frac{4(k+1)}{1+2k}} T^{-\frac{4(k+1)}{1+2k}} + \frac{M^2}{6N^2}(k+1)(1+2k)^{-\frac{(2k+3)}{1+2k}} T^{-\frac{4(k+1)}{1+2k}}, \tag{56}$$

$$C^{14}{}_{14} = \frac{M^2}{6N^2}(1+2k)^{-\frac{4(k+1)}{1+2k}} T^{-\frac{4(k+1)}{1+2k}} + \frac{M^2}{6N^2}(1-2k)(1+2k)^{-\frac{(2k+3)}{1+2k}} T^{-\frac{4(k+1)}{1+2k}}. \tag{57}$$

Here we find that

$$C^{12}{}_{12} + C^{13}{}_{13} + C^{14}{}_{14} = 0. \tag{58}$$

The model represents shearing, non-rotating and Petrov Type I non-degenerate universe in general, in which the flow is geodetic. In absence of shear viscosity, the model (44) starts expanding with a big bang at $T = 0$ when $k < -2$. The model starts expanding at $T = 0$ and goes on expanding till $T \to \infty$ when $-1 < k < -\frac{1}{2}$. When $T \to 0$ then $\rho \to \infty$ and $p \to \infty$ if $-1 < k < -\frac{1}{2}$. When $T \to \infty$ then $\rho \to 0$, $p \to 0$ if $-1 < k < -\frac{1}{2}$. Since $\lim_{T\to 0} \frac{\sigma}{\theta} \neq 0$, the models do not approach isotropy for large values of T. There is a Point Type singularity in the models at $T = 0$ when $k > 0$.

5 Concluding Remarks

In this paper we have described a new class of plane symmetric cosmological models of the universe with a viscous fluid as the source of matter. Generally, the models represent expanding, shearing, non-rotating and Petrov type non-degenerate universe in which the flow vector is geodetic. In all these models, we observe that they do not approach isotropy for large values of time. The solutions obtained in this paper generalize the previous solutions recently obtained by Singh and Yadav [50].

We shall find a mechanism in given situations where all these models have distinguish features based on macro details assign to them. As a preliminary thing models in Sects. 3.1 and 3.2 are more feasible class of models of bulk viscous fluid. Model in Sect. 4.1 seems to be of more wider class in comparison to Sects. 3.1 and 3.2. So it may be possible to incorporate features discussed as toy models in previous sections. This can only be quantified after details analysis which would be reported elsewhere. The cosmological terms in all models given in Sects. 3.1, 3.2 and 4.1 are decreasing function of time, and approach a small positive value as time increases (i.e., the present epoch). The values of Λ for these models are found to be small and a positive, which are supported by the results from recent supernovae Ia observations [11, 12, 15, 27–29, 44, 45, 49]. We find for wider class of parameters explore in the set up the density ρ and cosmological term Λ become negative in model in Sect. 4.2. We also find that for some cases the Λ has large oscillatory behaviour in the initial stage which do not seem to be any physical situation. Hence it will not be discussed anywhere. For the model described in Sect. 4.2, we observe that ρ and Λ both remain negative throughout the evolution. We have examined this scenario for wide range of parameters and come to the conclusion that models following $\xi = \xi_0 \rho^n$ $(n \geq 1)$ may not be physical in this set up.

The effect of bulk viscosity is to produce a change in perfect fluid and hence exhibit essential influence on the character of the solution. We also observe here that Murphy's conclusion [20] about the absence of a big bang type singularity in the infinite past in models with bulk viscous fluid, in general, is not true. The results obtained by Myung and Cho [21] also show that, it is, in general, not valid, since for some cases big bang singularity occurs in finite past.

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